

# Multicomponent theory of buoyancy instabilities in magnetized plasmas: The case of magnetic field parallel to gravity

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## ABSTRACT

We investigate electromagnetic buoyancy instabilities of the electron-ion plasma with the heat flux based on not the magnetohydrodynamic (MHD) equations, but using the multicomponent plasma approach when the momentum equations are solved for each species. We consider a geometry in which the background magnetic field, gravity, and stratification are directed along one axis. The nonzero background electron thermal flux is taken into account. Collisions between electrons and ions are included in the momentum equations. No simplifications usual for the one-fluid MHD-approach in studying these instabilities are used. We derive a simple dispersion relation, which shows that the thermal flux perturbation generally stabilizes an instability for the geometry under consideration. This result contradicts to conclusion obtained in the MHD-approach. We show that the

reason of this contradiction is the simplified assumptions used in the MHD analysis of buoyancy instabilities and the role of the longitudinal electric field perturbation which is not captured by the ideal MHD equations. Our dispersion relation also shows that the medium with the electron thermal flux can be unstable, if the temperature gradients of ions and electrons have the opposite signs. The results obtained can be applied to the weakly collisional magnetized plasma objects in laboratory and astrophysics.

**Keywords** convection - instabilities - magnetic fields - plasmas - waves

## 1. INTRODUCTION

Thermal effects resulting in instabilities, transport, heating, structures forming, and so on play an important role in dynamics of different plasma objects in laboratory, space, and astrophysics. For example, the ion-temperature-gradient-driven modes (Kadomtsev and Pogutse 1965) are used for explaining anomalous transport in tokamak plasma experiments (Dimits et al. 2000; Garbet 2001). Thermal conductivity influences on the Rayleigh-Taylor instability in inertial fusion (Betti et al. 1998; Canaud et al. 2004; Lindl et al. 2004), on the surface of the Sun (Isobe et al. 2005), in supernova (Fryxell et al. 1991), and other astrophysical objects. Thermally stratified fluids can be buoyantly unstable in the gravitational field. In astrophysics, this process may, for example, operates in the stellar interiors (Schwarzschild 1958), accretion disks (Balbus 2000, 2001), neutron stars (Chang and Quataert 2010), hot accretion flows (Narayan et al. 2000, 2002), galaxy clusters, and intracluster medium (ICM) (Quataert 2008; Parrish et al. 2009; Ren et al. 2009, 2010a). Analogous instabilities also exist in the neutral atmosphere of the Earth and ocean (Gossard and Hooke 1975; Pedlosky 1982). Diversity of environments in which buoyancy (or convective) instabilities may have the significant role, leading to turbulence and anomalous energy and matter transport, makes these instabilities an important object for analytical and numerical explorations.

The crucial role of convection in the transport of energy, for example, in stellar interiors is a well-known physical process (Schwarzschild 1958). However, theoretical efforts to understand convective energy transport in the dilute and hot plasmas such as galaxies clusters and ICM (Sarazin 1988) have lead to some results over recent years. As it is known, majority of the mass of a cluster of galaxies is in the dark matter. However, around 1/6 of its mass consists of a hot, magnetized, and low density plasma known as ICM. The electron number density is  $n_e \simeq 10^{-3}$  to  $10^{-1} \text{ cm}^{-3}$  at the central parts of ICM. The

electron temperature  $T_e$  is measured of the order of  $1 - 15$  keV, though the ion temperature  $T_i$  has not yet been measured directly (Fabian et al. 2006; Sanders et al. 2010). The magnetic field in ICM is estimated to be in the range  $0.1 - 10 \mu\text{G}$  depending on where the measurement is made (Carilli and Taylor 2002). This implies a dynamically weak magnetic field with  $\beta = 8\pi n_e T_e / B^2 \approx 200 - 2000$ . Under conditions given above, the Larmor radius of electrons and ions ( $T_i \sim T_e$ ) is many orders of magnitude smaller than the mean free path. Thus, the ICM is classified as a weakly collisional plasma (Carilli and Taylor 2002) possessing anisotropic transport due to the magnetic field.

In recent past, for such plasmas in the framework of the ideal MHD supplemented by an anisotropic heat flux along the magnetic field, there were found some new convective instabilities for the case when a heat flux plays the significant role (Balbus 2000, 2001; Quataert 2008; Ren et al. 2009, 2010a, 2010b). One of these instabilities, at the absence of the background thermal flux, has been shown to arise when the temperature increases in the direction of gravity which is perpendicular to the background magnetic field. This is so-called the magnetothermal instability (MTI) (Balbus 2000, 2001). The other instability, the heat buoyancy instability (HBI) (Quataert 2008), has been found to arise at the presence of the background heat flux when the temperature decreases along gravity parallel to the magnetic field. Anisotropic dissipative effects have been included by Ren et al. (2010a, 2010b).

Theoretical models applied for study of buoyancy instabilities in astrophysical objects with a heat flux are based on the one-fluid ideal (Balbus 2000, 2001; Quataert 2008; Ren et al. 2009; Chang and Quataert 2010) and nonideal (Ren et al. 2010a, 2010b) MHD equations. By using of these equations one can comparatively easily to consider any problems. However, the ideal MHD does not capture some important effects which can be taken into account by using a multi-fluid plasma approach. One of such effects is the

nonzero longitudinal electric field perturbation along the background magnetic field. An importance of involving this component due to multi-fluid effects and shortcomings of the ideal MHD were emphasized, for example, for the acceleration of solar flare electrons by inertial Alfvén waves (McClements and Fletcher 2009), at consideration of structures of electromagnetic fields and plasma flows in pulsar magnetosphere (Kojima and Oogi 2009), for the acceleration of relativistic ions, electrons, and positrons in shock waves (Takahashi et al. 2009), and in a gyrofluid description of Alfvénic turbulence (Bian and Kontar 2010). As we show here, the contribution of currents due to this (small in the present case) parallel electric field to the dispersion relation can be of the same order of magnitude as that due to transverse electric field components. Besides, the MHD equations do not take into account the existence of various charged and neutral species with different masses and electric charges and their collisions between each others and therefore can not be applied to multicomponent systems. In some cases, the standard methods used in the MHD lead to conclusions that are different from those obtained by the method using the electric field perturbations (the **E**-approach). One such an example concerning the contribution of the electron-ion collisions to the dispersion relation for the MHD waves in the two-component magnetized plasma was considered by Nekrasov (2009c). A multicomponent approach has been used in (Nekrasov 2008, 2009a, 2009b, 2009c, 2009d), where the streaming instabilities of rotating astrophysical objects (accretion disks, molecular clouds and so on) have been investigated.

A study of buoyancy instabilities with the electron heat flux by the multicomponent **E**-approach has been performed by Nekrasov and Shadmehri (2010). The geometry has been considered in which the gravity is perpendicular to the background magnetic field and the background heat flux is absent. Solution of the dispersion relation obtained in this paper differs from solution of the same problem found from the ideal MHD (Balbus 2000).

In this paper, we apply a multicomponent approach to study buoyancy instabilities in magnetized electron-ion plasmas with the background electron thermal flux. We consider the geometry in which the gravity, stratification, background magnetic field and thermal flux are all directed along one ( $z$ -) axis. For generality, we include collisions between electrons and ions in the momentum equations. At the consideration of the perturbed heat flux, we adopt that cyclotron frequencies of species are much larger than their collision frequencies. Such conditions are typical for many laboratory, space, and astrophysical plasmas. In this case, the heat flux is anisotropic and directed along the magnetic field lines (Braginskii 1965). However in other respects, the relation between the cyclotron and collision frequencies is arbitrary in the general expressions for the perturbed values. We derive the dispersion relation for cases, in which the background heat flux is present or absent. This gives a possibility to compare these two cases. Solutions of the dispersion relation are discussed.

The paper is organized as follows. In Sect. 2, the fundamental equations are given. An equilibrium state is considered in Sect. 3. Perturbed ion velocity, number density, and thermal pressure are obtained in Sect. 4. In Sect. 5, we consider the perturbed velocity and temperature for electrons. Components of the dielectric permeability tensor are found in Sect. 6. Dispersion relation is derived and considered in the collisionless and collisional cases in Sect. 7. Discussion of the results obtained and comparison with the MHD results are provided in Sect. 8. In Sect. 9, we give conclusive remarks.

## 2. BASIC EQUATIONS

We start with the following equations for ions:

$$\frac{\partial \mathbf{v}_i}{\partial t} = -\frac{\nabla p_i}{m_i n_i} + \mathbf{g} + \frac{q_i}{m_i} \mathbf{E} + \frac{q_i}{m_i c} \mathbf{v}_i \times \mathbf{B} - \nu_{ie} (\mathbf{v}_i - \mathbf{v}_e), \quad (1)$$

the momentum equation,

$$\frac{\partial n_i}{\partial t} + \nabla \cdot n_i \mathbf{v}_i = 0, \quad (2)$$

the continuity equation, and

$$\frac{\partial p_i}{\partial t} + \mathbf{v}_i \cdot \nabla p_i + \gamma p_i \nabla \cdot \mathbf{v}_i = 0, \quad (3)$$

the pressure equation. The corresponding equations for electrons are:

$$\mathbf{0} = -\frac{\nabla p_e}{n_e} + q_e \mathbf{E} + \frac{q_e}{c} \mathbf{v}_e \times \mathbf{B} - m_e \nu_{ei} (\mathbf{v}_e - \mathbf{v}_i), \quad (4)$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot n_e \mathbf{v}_e = 0, \quad (5)$$

$$\frac{\partial p_e}{\partial t} + \mathbf{v}_e \cdot \nabla p_e + \gamma p_e \nabla \cdot \mathbf{v}_e = \lambda - (\gamma - 1) \nabla \cdot \mathbf{q}_e, \quad (6)$$

$$\frac{\partial T_e}{\partial t} + \mathbf{v}_e \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot \mathbf{v}_e = \frac{\lambda}{n_e} - (\gamma - 1) \frac{1}{n_e} \nabla \cdot \mathbf{q}_e, \quad (7)$$

the temperature equation, where  $\mathbf{q}_e$  is the electron heat flux (Braginskii 1965). We neglect inertia of the electrons. In (1)-(7),  $q_j$  and  $m_j$  are the charge and mass of species  $j = i, e$ ,  $\mathbf{v}_j$  is the hydrodynamic velocity,  $n_j$  is the number density,  $p_j = n_j T_j$  is the thermal pressure,  $T_j$  is the temperature,  $\nu_{ie}$  ( $\nu_{ei}$ ) is the collision frequency of ions (electrons) with electrons (ions),  $\mathbf{g}$  is gravity,  $\mathbf{E}$  and  $\mathbf{B}$  are the electric and magnetic fields,  $c$  is the speed of light in vacuum, and  $\gamma$  is the adiabatic constant. At the consideration of the electron heat flux, we will assume the electrons to be magnetized when their cyclotron frequency  $\omega_{ce} = q_e B / m_e c \gg \nu_{ee}(\nu_{ei})$ , where  $\nu_{ee}(\nu_{ei})$  is the electron-electron (ion) collision frequency. In this case, the electron thermal flux is mainly directed along the magnetic field,

$$\mathbf{q}_e = -\chi_e \mathbf{b} (\mathbf{b} \cdot \nabla) T_e, \quad (8)$$

where  $\chi_e$  is the electron thermal conductivity coefficient and  $\mathbf{b} = \mathbf{B}/B$  is the unit vector along the magnetic field (Braginskii 1965). However in the momentum equations (1) and (4), we keep for generality an arbitrary relation between  $\omega_{ci}(\omega_{ce})$  and  $\nu_{ie}(\nu_{ei})$  ( $\omega_{ci} = q_i B / m_i c$

and  $\nu_{ie}$  are the ion cyclotron and ion-electron collision frequencies, respectively), having in mind that some expressions obtained below can be applied for collisional objects. The term  $\lambda$  compensates the temperature change as a result of the equilibrium heat flux. We take only into account the electron thermal conductivity by (8), because the corresponding ion conductivity is considerably smaller (Braginskii 1965). For generality, we assume the electron and ion temperatures to be different. However, we do not involve, for simplicity, the terms describing the energy exchange between ions and electrons in (3), (6), and (7). Thus, our treatment is available for cases in which such an exchange is not effective or when there is a strong temperature coupling of species. In the last case, one can set  $T_i \simeq T_e$ . This issue is considered in more detail in Sect. 8.

Electromagnetic equations are Faraday's law

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (9)$$

and Ampere's law

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}, \quad (10)$$

where  $\mathbf{j} = \sum_j q_j n_j \mathbf{v}_j$ . We consider the wave processes with typical timescales much larger than the time the light spends to cover the wavelength of perturbations. In this case, one can neglect the displacement current in (10) that results in quasineutrality both in electromagnetic and purely electrostatic perturbations. The magnetic field  $\mathbf{B}$  includes the background magnetic field  $\mathbf{B}_0$ , the magnetic field  $\mathbf{B}_{0cur}$  of the background current (when it presents), and the perturbed magnetic field.

### 3. EQUILIBRIUM STATE

At first, we consider an equilibrium state. We assume that background velocities are absent. In this paper, we study configuration in which the background magnetic field,



gravity, and stratification are directed along the  $z$ -axis. Let, for definiteness,  $\mathbf{g}$  be  $\mathbf{g} = -\mathbf{z}g$ , where  $g > 0$  and  $\mathbf{z}$  is the unit vector along the  $z$ -direction. Then, (1) and (4) give

$$g_i = -\frac{1}{m_i n_{i0}} \frac{\partial p_{i0}}{\partial z} = g - \frac{q_i}{m_i} E_0, \quad (11)$$

$$g_e = -\frac{1}{m_i n_{e0}} \frac{\partial p_{e0}}{\partial z} = \frac{q_i}{m_i} E_0, \quad (12)$$

where (and below) the index 0 denotes equilibrium values. Here and below, we assume that  $q_i = -q_e$ . For convenience of notations, we do not use that  $n_{i0} = n_{e0}$  for the two-component plasma up to a point where it will be necessary. We see that equilibrium distributions of ions and electrons influence on each other through the background electric field  $E_0$ . In the case  $n_{i0} = n_{e0}$  and  $T_{i0} = T_{e0}$ , we have  $g_i = g_e = g/2$ . Thus, we obtain  $E_0 = m_i g / 2q_i$ . The presence of the third component, for example, of the cold dust grains with the charge  $q_d$  and mass  $m_d \gg m_i$  results in other value of  $E_0 = m_d g / q_d$ . In this case, the ions and electrons are in equilibrium under the action of the thermal pressure and equilibrium electric field, being  $g_i \simeq -g_e$ , if  $q_i m_d \gg q_d m_i$ .

#### 4. LINEAR ION PERTURBATIONS

Let us write (1)-(3) for ions in the linear approximation,

$$\frac{\partial \mathbf{v}_{i1}}{\partial t} = -\frac{\nabla p_{i1}}{m_i n_{i0}} + \frac{\nabla p_{i0}}{m_i n_{i0}} \frac{n_{i1}}{n_{i0}} + \mathbf{F}_{i1} + \frac{q_i}{m_i c} \mathbf{v}_{i1} \times \mathbf{B}_0, \quad (13)$$

$$\frac{\partial n_{i1}}{\partial t} + v_{i1z} \frac{\partial n_{i0}}{\partial z} + n_{i0} \nabla \cdot \mathbf{v}_{i1} = 0, \quad (14)$$

$$\frac{\partial p_{i1}}{\partial t} + v_{i1z} \frac{\partial p_{i0}}{\partial z} + \gamma p_{i0} \nabla \cdot \mathbf{v}_{i1} = 0, \quad (15)$$

where

$$\mathbf{F}_{i1} = \frac{q_i}{m_i} \mathbf{E}_1 - \nu_{ie} (\mathbf{v}_{i1} - \mathbf{v}_{e1}), \quad (16)$$

and the index 1 denotes the perturbed variables. Below, we solve these equations to find the perturbed velocity of ions in an inhomogeneous medium.

#### 4.1. Perturbed velocity of ions

Applying the operator  $\partial/\partial t$  to (13) and using (14) and (15), we obtain equation containing only the ion velocity

$$\frac{\partial^2 \mathbf{v}_{i1}}{\partial t^2} = -g_i \nabla v_{i1z} + \frac{1}{m_i n_{i0}} [(\gamma - 1) (\nabla p_{i0}) + \gamma p_{i0} \nabla] \nabla \cdot \mathbf{v}_{i1} + \frac{\partial \mathbf{F}_{i1}}{\partial t} + \frac{q_i}{m_i c} \frac{\partial \mathbf{v}_{i1}}{\partial t} \times \mathbf{B}_0. \quad (17)$$

From this equation, we can find solutions for the components of  $\mathbf{v}_{i1}$ . For simplicity, we assume that  $\partial/\partial x = 0$  because a system is symmetric in the transverse direction relative to the  $z$ -axis. The  $x$ -component of (17) has the simple form

$$\frac{\partial v_{i1x}}{\partial t} = F_{i1x} + \omega_{ci} v_{i1y}. \quad (18)$$

Here  $\omega_{ci} = q_i B_0 / m_i c$ . For the  $y$ -component of (17), we obtain:

$$\frac{\partial^2 v_{i1y}}{\partial t^2} = -g_i \frac{\partial v_{i1z}}{\partial y} + c_{si}^2 \frac{\partial}{\partial y} \nabla \cdot \mathbf{v}_{i1} + \frac{\partial F_{i1y}}{\partial t} - \omega_{ci} \frac{\partial v_{i1x}}{\partial t}, \quad (19)$$

where,  $c_{si} = (\gamma T_{i0} / m_i)^{1/2}$  is the ion sound velocity. Using (18), equation for  $v_{i1y}$  is given from (19) as follows

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) v_{i1y} - Q_{i1y} = \frac{\partial P_{i1}}{\partial y}. \quad (20)$$

Then from (18), we obtain

$$\frac{\partial}{\omega_{ci} \partial t} \left[ \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) v_{i1x} - Q_{i1x} \right] = \frac{\partial P_{i1}}{\partial y}. \quad (21)$$

In (20) and (21), we have introduced the following notations:

$$P_{i1} = -g_i v_{i1z} + c_{si}^2 \nabla \cdot \mathbf{v}_{i1}, \quad (22)$$

$$Q_{i1x} = \omega_{ci} F_{i1y} + \frac{\partial F_{i1x}}{\partial t}, \quad (23)$$

$$Q_{i1y} = -\omega_{ci} F_{i1x} + \frac{\partial F_{i1y}}{\partial t}. \quad (24)$$

The value  $P_{i1}$  defines the ion pressure perturbation (see 15). We see from (20) and (21) that the thermal pressure effect on the velocity  $v_{i1x}$  is much larger than that on  $v_{i1y}$  when  $\partial/\partial t \ll \omega_{ci}$ .

The  $z$ -component of (17) takes the form

$$\frac{\partial}{\partial t} \left( \frac{\partial v_{i1z}}{\partial t} - F_{i1z} \right) = -g_i \frac{\partial v_{i1z}}{\partial z} + \left[ (1 - \gamma) g_i + c_{si}^2 \frac{\partial}{\partial z} \right] \nabla \cdot \mathbf{v}_{i1}. \quad (25)$$

To obtain equation only for  $v_{i1z}$ , we need to express  $\nabla \cdot \mathbf{v}_{i1}$  through  $v_{i1z}$ . Differentiating (20) with respect to  $y$  and using expression (22), we find

$$L_1 \nabla \cdot \mathbf{v}_{i1} = L_2 v_{i1z} + \frac{\partial Q_{i1y}}{\partial y}, \quad (26)$$

where the following operators are introduced:

$$L_1 = \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 - c_{si}^2 \frac{\partial^2}{\partial y^2}, \quad (27)$$

$$L_2 = \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial}{\partial z} - g_i \frac{\partial^2}{\partial y^2}. \quad (28)$$

We can derive equation for the longitudinal velocity  $v_{i1z}$ , substituting  $\nabla \cdot \mathbf{v}_{i1}$  found from (26) into (25),

$$L_3 v_{i1z} = L_1 \frac{\partial F_{i1z}}{\partial t} + L_4 \frac{\partial Q_{i1y}}{\partial y}, \quad (29)$$

where operators  $L_3$  and  $L_4$  have the form

$$\begin{aligned} L_3 = & \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial^2}{\partial t^2} - c_{si}^2 \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \frac{\partial^2}{\partial t^2} - c_{si}^2 \omega_{ci}^2 \frac{\partial^2}{\partial z^2} \\ & + \gamma g_i \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial}{\partial z} + c_{si}^2 \frac{\partial L_1}{L_1 \partial z} L_2 + (1 - \gamma) g_i^2 \frac{\partial^2}{\partial y^2}, \end{aligned} \quad (30)$$

$$L_4 = (1 - \gamma) g_i + c_{si}^2 \left( \frac{\partial}{\partial z} - \frac{\partial L_1}{L_1 \partial z} \right). \quad (31)$$

For obtaining expression (30), we have used expressions (27) and (28).

It is easy to see that at the absence of the background magnetic field and without taking into account electromagnetic perturbations and collisions with electrons (the right hand-side of 29), equation  $L_3 v_{i1z} = 0$  describes the ion sound and internal gravity waves. In this case, a sum of the last two terms on the right hand-side of expression (30) is equal to  $-c_{si}^2 \omega_{bi}^2 \frac{\partial^2}{\partial y^2}$ , where  $\omega_{bi}$  is the (ion) Brunt-Väisälä frequency equal to

$$\omega_{bi}^2 = \frac{g_i}{c_{si}^2} \left[ (\gamma - 1) g_i + \frac{\partial c_{si}^2}{\partial z} \right]. \quad (32)$$

Thus, we have obtained a result corresponding to perturbations in the neutral atmosphere (Gossard and Hooke 1975). However, we see that the existence of the background magnetic field considerably modifies the operator  $L_3$ . We note that the right hand-side of (29) describes a connection between ions and electrons through the electric field  $\mathbf{E}_1$  and collisions.

#### 4.2. Specific case for ions

So far, we did not make any simplifications and all the equations and expressions obtained above are given in their general form. This permits us to investigate different limiting cases. Further, we consider perturbations with a frequency much lower than the ion cyclotron frequency and the transverse wavelengths much larger than the ion Larmor radius. Such perturbations are of interest for both laboratory and astrophysical plasmas. Besides, we investigate a part of the frequency spectrum in the region lower than the ion sound frequency. Thus, we set

$$\omega_{ci}^2 \gg \frac{\partial^2}{\partial t^2}, c_{si}^2 \frac{\partial^2}{\partial y^2}; c_{si}^2 \frac{\partial^2}{\partial z^2} \gg \frac{\partial^2}{\partial t^2}. \quad (33)$$

In this case, operators (27), (28), (30), and (31) take the form

$$\begin{aligned} L_1 &\simeq \omega_{ci}^2, L_2 \simeq \omega_{ci}^2 \frac{\partial}{\partial z}, \\ L_3 &= -\omega_{ci}^2 \left[ \left( c_{si}^2 \frac{\partial}{\partial z} - \gamma g_i \right) \frac{\partial}{\partial z} - \frac{\partial^2}{\partial t^2} \right], \\ L_4 &= (1 - \gamma) g_i + c_{si}^2 \frac{\partial}{\partial z}. \end{aligned} \quad (34)$$

Also, an additional condition

$$\omega_{ci}^2 \frac{\partial^2}{\partial t^2} \gg c_{si}^2 \frac{\partial c_{si}^2}{\partial z} \frac{\partial^3}{\partial y^2 \partial z} \quad (35)$$

must be satisfied for operator  $L_3$  to have a given form (34). The small corrections in operators  $L_3$  and  $L_4$  are needed to be kept because some main terms in expressions for the ion and electron velocities are equal to each other (see below). Therefore, when calculating the electric current, these main terms will be canceled and small corrections to velocities will only contribute to the current. The applicability of condition (35) and other conditions used below will be discussed in Sect. 8.

For cases represented by inequalities (33) and (35) when the operators  $L_i$ ,  $i = 1, 2, 3, 4$ , have the form (34), equations for  $v_{i1z}$  and  $\nabla \cdot \mathbf{v}_{i1}$  become

$$\left[ \left( c_{si}^2 \frac{\partial}{\partial z} - \gamma g_i \right) \frac{\partial}{\partial z} - \frac{\partial^2}{\partial t^2} \right] v_{i1z} = -\frac{\partial F_{i1z}}{\partial t} - \left[ (1 - \gamma) g_i + c_{si}^2 \frac{\partial}{\partial z} \right] \frac{\partial Q_{i1y}}{\omega_{ci}^2 \partial y}, \quad (36)$$

$$\nabla \cdot \mathbf{v}_{i1} \simeq \frac{\partial v_{i1z}}{\partial z} + \frac{\partial Q_{i1y}}{\omega_{ci}^2 \partial y}. \quad (37)$$

### 4.3. Ion perturbations in the Fourier transformation

Calculations show that some main terms in expressions for  $v_{i1z}$  (when calculating the current),  $\nabla \cdot \mathbf{v}_{i1}$  and  $P_{i1}$  are canceled. Therefore, the small terms proportional to inhomogeneity must be taken into account. To do this correctly, we can not apply the Fourier transformation to (36) and (37) to find the variable  $P_{i1}$ . However, firstly, we should

apply the operator  $\partial/\partial z$  to this variable for using (36). It is analogous to obtaining the term  $\partial c_s^2/\partial z$  in expression (32) for the Brunt-Väisälä frequency. After that, we can apply the Fourier transformation in a local approximation assuming the linear perturbations to be proportional to  $\exp(i\mathbf{k}\mathbf{r}-i\omega t)$ . As a result, we obtain for the Fourier-components  $v_{i1zk}$ ,  $\mathbf{k} \cdot \mathbf{v}_{i1k}$ , and  $P_{i1k}$ , where  $k = (\mathbf{k}, \omega)$ , the following expressions:

$$v_{i1zk} = -i \frac{\omega}{k_z^2 c_{si}^2} \left( 1 - i \frac{\gamma g_i}{k_z c_{si}^2} \right) F_{i1zk} - \frac{k_y}{k_z \omega_{ci}^2} \left( 1 - i \frac{g_i}{k_z c_{si}^2} \right) Q_{i1yk}, \quad (38)$$

$$\mathbf{k} \cdot \mathbf{v}_{i1k} = -i \frac{\omega}{k_z c_{si}^2} \left( 1 - i \frac{\gamma g_i}{k_z c_{si}^2} \right) F_{i1zk} + i \frac{k_y}{k_z} \frac{g_i}{c_{si}^2 \omega_{ci}^2} Q_{i1yk}, \quad (39)$$

$$\begin{aligned} P_{i1k} = & \frac{\omega}{k_z} F_{i1zk} - i \frac{\omega}{k_z^2 c_{si}^2} \left[ (\gamma - 1) g_i + \frac{\partial c_{si}^2}{\partial z} \right] F_{i1zk} \\ & + i \frac{k_y g_i}{k_z^2 c_{si}^2 \omega_{ci}^2} \left[ (\gamma - 1) g_i + \frac{\partial c_{si}^2}{\partial z} - \omega^2 \frac{c_{si}^2}{g_i} \right] Q_{i1yk}, \end{aligned} \quad (40)$$

where  $g_i/k_z c_{si}^2 \ll 1$ . In expressions (38) and (39), we have omitted additional small terms at  $Q_{i1yk}$  which are needed for calculation of  $P_{i1k}$ .

When calculating the current along the  $z$ -axis, the main term  $\sim Q_{i1yk}$  in (38) will be canceled with the corresponding electron term. The contribution of the first term  $\sim F_{i1zk}$  to this current has, as we will see below, the same order of magnitude for the buoyancy instabilities as contribution of the term  $\sim g_i Q_{i1yk}$ , i.e.  $F_{i1zk} \sim (k_y g_i / \omega \omega_{ci}^2) Q_{i1yk}$ . The same relates to expressions (39) and (40). Thus, the longitudinal electric field perturbation  $E_{1z}$  containing in  $F_{i1z}$  must be taken into account. However, in the ideal MHD, this field is absent. We see from expressions (38) and (39) that  $\nabla \cdot \mathbf{v}_{i1} \sim (g_i/c_{si}^2) v_{i1z}$ . This relation is the same as that for internal gravity waves in the Earth's atmosphere (see, e.g., Nekrasov 1994). Using expression (40), we can find velocities  $v_{i1yk}$  and  $v_{i1xk}$  from (20) and (21), correspondingly.

#### 4.4. Perturbed ion number density and pressure

It is followed from above that  $\nabla \cdot \mathbf{v}_{i1} \sim v_{i1z}/H$ , where  $H$  is the inhomogeneity scale height ( $H \sim c_{si}^2/g_i$ ). Thus, the last two terms in (14) and (15) are of the same order of magnitude. Let us find the perturbed ion number density and pressure in the Fourier-representation. From (14), (38) and (39), we obtain

$$\frac{n_{i1k}}{n_{i0}} = -i \frac{1}{k_z c_{si}^2} F_{i1zk} - i \frac{k_y}{k_z c_{si}^2 \omega \omega_{ci}^2} \left[ (\gamma - 1) g_i + \frac{\partial c_{si}^2}{\partial z} \right] Q_{i1yk}. \quad (41)$$

Equation (15) gives  $\partial p_{i1}/\partial t = -m_i n_{i0} P_{i1}$ . Thus, we have, using (40),

$$\frac{p_{i1k}}{p_{i0}} = -i \frac{\gamma}{k_z c_{si}^2} F_{i1zk} + \frac{\gamma k_y g_i}{k_z^2 c_{si}^4 \omega \omega_{ci}^2} \left[ (\gamma - 1) g_i + \frac{\partial c_{si}^2}{\partial z} - \omega^2 \frac{c_{si}^2}{g_i} \right] Q_{i1yk}. \quad (42)$$

Comparing (41) and (42), we see that the relative perturbation of the pressure due to the transverse electric force  $Q_{i1yk}$  is much smaller than the relative perturbation of the number density. However, these relative perturbations as a result of the action of the longitudinal electric force  $F_{i1zk}$  have the same order of magnitude (see Sect. 4.3). Thus,  $p_{i1k}/p_{i0} \sim n_{i1k}/n_{i0}$ . This result contradicts a supposition  $p_{i1k}/p_{i0} \ll n_{i1k}/n_{i0}$  adopted in the MHD analysis of buoyancy instabilities (Balbus 2000, 2001; Quataert 2008). From results obtained below, it is followed that, as we already have noted above, the both terms on the right hand-side of (41) have the same order of magnitude.

### 5. LINEAR ELECTRON PERTURBATIONS

Equations for the electrons in the linear approximation are the following:

$$\mathbf{0} = -\frac{\nabla p_{e1}}{n_{e0}} + \frac{\nabla p_{e0}}{n_{e0}} \frac{n_{e1}}{n_{e0}} + \mathbf{F}_{e1} + \frac{q_e}{c} \mathbf{v}_{e1} \times \mathbf{B}_0, \quad (43)$$

$$\frac{\partial n_{e1}}{\partial t} + v_{e1z} \frac{\partial n_{e0}}{\partial z} + n_{e0} \nabla \cdot \mathbf{v}_{e1} = 0, \quad (44)$$

$$\frac{\partial p_{e1}}{\partial t} + v_{e1z} \frac{\partial p_{e0}}{\partial z} + \gamma p_{e0} \nabla \cdot \mathbf{v}_{e1} = -(\gamma - 1) \nabla \cdot \mathbf{q}_{e1}, \quad (45)$$

$$\frac{\partial T_{e1}}{\partial t} + v_{e1z} \frac{\partial T_{e0}}{\partial z} + (\gamma - 1) T_{e0} \nabla \cdot \mathbf{v}_{e1} = -(\gamma - 1) \frac{1}{n_{e0}} \nabla \cdot \mathbf{q}_{e1}, \quad (46)$$

$$\mathbf{q}_{e1} = -\mathbf{b}_1 \chi_{e0} \frac{\partial T_{e0}}{\partial z} - \mathbf{b}_0 \chi_{e0} \frac{\partial T_{e1}}{\partial z} - \mathbf{b}_0 \chi_{e1} \frac{\partial T_{e0}}{\partial z}, \quad (47)$$

$$\mathbf{F}_{e1} = q_e \mathbf{E}_1 - m_e \nu_{ei} (\mathbf{v}_{e1} - \mathbf{v}_{i1}). \quad (48)$$

In (47),  $\chi_{e1} = 5\chi_{e0}T_{e1}/2T_{e0}$  is the perturbation of the thermal flux conductivity coefficient  $\chi_e$  which is proportional to  $T_e^{5/2}$  (Spitzer 1962; Braginskii 1965). The perturbation of the unit magnetic vector  $\mathbf{b}_1$  is equal to  $b_{1x,y} = B_{1x,y}/B_0$  and  $b_{1z} = 0$ . The thermal flux in equilibrium is  $\mathbf{q}_{e0} = -\mathbf{b}_0 \chi_{e0} \frac{\partial T_{e0}}{\partial z}$ .

We have seen above at consideration of the ion perturbations that the terms  $\sim 1/H^2$  are needed to be kept (see the last term in 40). Therefore, we also keep such terms for the electrons.

### 5.1. Equation for the electron temperature perturbation

Let us now find equation for the electron temperature perturbation. Expression  $\nabla \cdot \mathbf{q}_{e1}$ , where  $\mathbf{q}_{e1}$  is defined by (47), is given by

$$\nabla \cdot \mathbf{q}_{e1} = \frac{\partial q_{e1y}}{\partial y} + \frac{\partial q_{e1z}}{\partial z} = -\chi_{e0} \frac{\partial T_{e0}}{\partial z} \frac{1}{B_0} \frac{\partial B_{1y}}{\partial y} - \chi_{e0} \frac{\partial^2 T_{e1}}{\partial z^2} - 2 \frac{\partial \chi_{e0}}{\partial z} \frac{\partial T_{e1}}{\partial z} - \frac{\partial^2 \chi_{e0}}{\partial z^2} T_{e1}. \quad (49)$$

Substituting this expression into (46), we obtain

$$D_1 T_{e1} = -v_{e1z} \frac{\partial T_{e0}}{\partial z} - (\gamma - 1) T_{e0} \nabla \cdot \mathbf{v}_{e1} + (\gamma - 1) \frac{\chi_{e0}}{n_{e0}} \frac{\partial T_{e0}}{\partial z} \frac{\partial B_{1y}}{B_0 \partial y}, \quad (50)$$

where the operator  $D_1$  is defined by

$$D_1 = \left[ \frac{\partial}{\partial t} - (\gamma - 1) \frac{1}{n_{e0}} \left( \chi_{e0} \frac{\partial^2}{\partial z^2} + 2 \frac{\partial \chi_{e0}}{\partial z} \frac{\partial}{\partial z} + \frac{\partial^2 \chi_{e0}}{\partial z^2} \right) \right]. \quad (51)$$



## 5.2. Perturbed velocity and temperature of electrons

We further find equations for components of the perturbed velocity of electrons. The  $x$ -component of (43) has a simple form, i.e.

$$v_{e1y} = -\frac{1}{m_e \omega_{ce}} F_{e1x}, \quad (52)$$

where  $\omega_{ce} = q_e B_0 / m_e c$ . Applying the operator  $\partial/\partial t$  to the  $y$ -component of (43) and using (45) and (49), we obtain

$$\begin{aligned} \frac{\partial}{\partial t} \left( v_{e1x} - \frac{1}{m_e \omega_{ce}} F_{e1y} \right) = & -\frac{1}{\omega_{ci}} \frac{\partial P_{e1}}{\partial y} - (\gamma - 1) \frac{\chi_{e0}}{m_e \omega_{ce} n_{e0}} \frac{\partial T_{e0}}{\partial z} \frac{\partial^2 B_{1y}}{B_0 \partial y^2} \\ & + \frac{1}{m_e \omega_{ce}} \left( D_1 - \frac{\partial}{\partial t} \right) \frac{\partial T_{e1}}{\partial y}. \end{aligned} \quad (53)$$

Here

$$P_{e1} = -g_e v_{e1z} + c_{se}^2 \nabla \cdot \mathbf{v}_{e1}, \quad (54)$$

where  $c_{se}^2 = \gamma p_{e0} / m_i n_{e0}$ . The variable  $P_{e1}$  is analogous to  $P_{i1}$  (see 22) and defines the electron pressure perturbation. But for electrons, their pressure perturbation is also affected by the thermal conductivity (see 45). The  $z$ -component of (43) takes the form

$$0 = -\frac{1}{n_{e0}} \frac{\partial p_{e1}}{\partial z} + \frac{1}{n_{e0}} \frac{\partial p_{e0}}{\partial z} \frac{n_{e1}}{n_{e0}} + F_{e1z}. \quad (55)$$

We can express  $\nabla \cdot \mathbf{v}_{e1}$  which is contained in (54) through  $v_{e1z}$ , using (52),

$$\nabla \cdot \mathbf{v}_{e1} = \frac{\partial v_{e1z}}{\partial z} - \frac{1}{m_e \omega_{ce}} \frac{\partial F_{e1x}}{\partial y}. \quad (56)$$

This expression for electrons is analogous to that for ions (see 37).

We further consider perturbations with the dynamical frequency  $\partial/\partial t$  satisfying the following conditions:

$$\frac{\chi_{e0}}{n_{e0}} \frac{\partial^2}{\partial z^2} \gg \frac{\partial}{\partial t} \gg \frac{1}{n_{e0}} \frac{\partial \chi_{e0}}{\partial z} \frac{\partial}{\partial z}. \quad (57)$$

The first inequality (57) means that the thermal conductivity is the dominant mode of the thermal transport (Balbus 2000; Quataert 2008). Under the second condition (57), we can

neglect the inhomogeneity and perturbation of the thermal flux conductivity coefficient in the temperature equation (50) (see 47 and 51). Obviously, the term proportional to  $\partial^2 \chi_{e0}/\partial z^2$  in expression (51) can also be neglected. In this case, the small correction proportional to  $\partial/\partial t$  in the temperature equation (50) which will be necessary for calculation of the electron velocity (see below) will be larger than that  $\sim \partial \chi_{e0}/\partial z$ . We further apply the operator  $\partial/\partial t$  to (55) and use (44), (45), (49), and (56). As a result, we obtain

$$\begin{aligned} \left( c_{se}^2 \frac{\partial}{\partial z} - \gamma g_e \right) \frac{\partial v_{e1z}}{\partial z} = & - \frac{\partial F_{e1z}}{m_i \partial t} + \left[ (1 - \gamma) g_e + c_{se}^2 \frac{\partial}{\partial z} \right] \frac{1}{m_e \omega_{ce}} \frac{\partial F_{e1x}}{\partial y} \\ & + (\gamma - 1) \frac{\chi_{e0}}{m_i n_{e0}} \left( \frac{\partial T_{e0}}{\partial z} \frac{1}{B_0} \frac{\partial^2 B_{1y}}{\partial y \partial z} + \frac{\partial^3 T_{e1}}{\partial z^3} \right). \end{aligned} \quad (58)$$

Equation for the temperature perturbation under conditions (57) has the form

$$\begin{aligned} \left[ (\gamma - 1) \frac{\chi_{e0}}{n_{e0}} \frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial t} \right] T_{e1} = & v_{e1z} \frac{\partial T_{e0}}{\partial z} + (\gamma - 1) T_{e0} \left( \frac{\partial v_{e1z}}{\partial z} - \frac{1}{m_e \omega_{ce}} \frac{\partial F_{e1x}}{\partial y} \right) \\ & - (\gamma - 1) \frac{\chi_{e0}}{n_{e0}} \frac{\partial T_{e0}}{\partial z} \frac{\partial B_{1y}}{B_0 \partial y}, \end{aligned} \quad (59)$$

where we have used (56).

To find equation for  $v_{e1z}$ , we substitute  $T_{e1}$  from (59) into (58). Taking into account a contribution of the term  $\partial T_{e1}/\partial t$  and carrying out some transformations, we obtain

$$\begin{aligned} \frac{\partial^3 v_{e1z}}{\partial z^3} = & - \frac{\partial^2 F_{e1z}}{T_{e0} \partial z \partial t} - \frac{n_{e0}}{\chi_{e0}} \left( \frac{\partial}{\partial z} \right)^{-1} \frac{\partial^2 F_{e1z}}{T_{e0} \partial t^2} + \frac{1}{m_e \omega_{ce}} \frac{\partial^3 F_{e1x}}{\partial y \partial z^2} \\ & + \frac{1}{c_{se}^2} \left( \gamma g_e + \frac{\partial c_{se}^2}{\partial z} \right) \frac{1}{m_e \omega_{ce}} \frac{\partial^2 F_{e1x}}{\partial y \partial z} - \frac{\partial T_{e0}}{T_{e0} \partial z} \frac{1}{B_0} \frac{\partial^2 B_{1y}}{\partial y \partial t}. \end{aligned} \quad (60)$$

The correction proportional to  $\partial F_{e1x}/\partial t$  is absent. The last term on the right hand-side of (60) is connected with the background electron thermal flux.

From (59) and (60), we can find equation for the temperature perturbation

$$\begin{aligned}
 (\gamma - 1) \frac{\chi_{e0}}{n_{e0}} \frac{\partial}{\partial z} \left( \frac{\partial^2 T_{e1}}{\partial z^2} + \frac{\partial T_{e0}}{\partial z} \frac{\partial B_{1y}}{B_0 \partial y} \right) &= \frac{\gamma T_{e0}}{c_{se}^2} \left[ (\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial z} \right] \frac{1}{m_e \omega_{ce}} \frac{\partial F_{e1x}}{\partial y} \\
 &- (\gamma - 1) \frac{\partial F_{e1z}}{\partial t} - \gamma \frac{n_{e0}}{\chi_{e0}} \left( \frac{\partial}{\partial z} \right)^{-2} \frac{\partial^2 F_{e1z}}{\partial t^2} \\
 &- \gamma \frac{\partial T_{e0}}{\partial z} \left( \frac{\partial}{\partial z} \right)^{-1} \frac{\partial^2 B_{1y}}{B_0 \partial y \partial t}.
 \end{aligned} \tag{61}$$

It is followed from results obtained below that all terms on the right-hand side of (61) (except the correction  $\sim \partial^2 F_{e1z}/\partial t^2$ ) have the same order of magnitude (see Sect. 4.3). The left-hand side of this equation is larger (see conditions 57). Thus, the temperature perturbation in the zero order of magnitude can be found by equaling the left part of (61) to zero. However, the right part of this equation is necessary for finding the transverse velocity perturbation  $v_{e1x}$  (see below).

To find the velocity  $v_{e1x}$ , we need to calculate the value  $P_{e1}$  (see 53 and 54). Performing calculations in the same way as that for ions (see Sect. 4.3), we obtain

$$\begin{aligned}
 c_{se}^2 \frac{\partial^2 P_{e1}}{\partial z^2} &= \left[ c_{se}^2 \frac{\partial}{\partial z} + (\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial z} \right] \left( -\frac{\partial F_{e1z}}{m_i \partial t} + \frac{\partial V_{e1}}{\partial z} \right) \\
 &+ g_e \left[ (\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial z} \right] \frac{1}{m_e \omega_{ce}} \frac{\partial F_{e1x}}{\partial y},
 \end{aligned} \tag{62}$$

where we have introduced notation connected with the thermal conductivity,

$$V_{e1} = (\gamma - 1) \frac{\chi_{e0}}{m_i n_{e0}} \left( \frac{\partial T_{e0}}{\partial z} \frac{1}{B_0} \frac{\partial B_{1y}}{\partial y} + \frac{\partial^2 T_{e1}}{\partial z^2} \right). \tag{63}$$

Equation (62) can be re-written in the form which is convenient for finding the velocity  $v_{e1x}$ . Using (61), we obtain

$$\begin{aligned}
 \frac{\partial^2}{\partial z^2} (P_{e1} - V_{e1}) &= -\frac{\partial^2 F_{e1z}}{m_i \partial z \partial t} - \frac{\gamma}{c_{se}^2} \left[ (\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial z} \right] \frac{\partial F_{e1z}}{m_i \partial t} \\
 &+ \frac{1}{c_{se}^2} \left[ (\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial z} \right] \left( \gamma g_e + \frac{\partial c_{se}^2}{\partial z} \right) \frac{1}{m_e \omega_{ce}} \frac{\partial F_{e1x}}{\partial y} \\
 &- \left[ (\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial z} \right] \frac{\partial T_{e0}}{T_{e0} \partial z} \left( \frac{\partial}{\partial z} \right)^{-1} \frac{\partial^2 B_{1y}}{B_0 \partial y \partial t}.
 \end{aligned} \tag{64}$$

It is easy to see that (53) has the form

$$\frac{\partial}{\partial t} \left( v_{e1x} - \frac{1}{m_e \omega_{ce}} F_{e1y} \right) = -\frac{1}{\omega_{ci}} \frac{\partial}{\partial y} (P_{e1} - V_{e1}). \quad (65)$$

Thus, the main contribution of the flux described by (63) does not influence on the electron dynamics. Applying operator  $\partial^2/\partial z^2$  to (65) and using (64), we find an equation for the perturbed velocity  $v_{e1x}$ .

## 6. FOURIER CURRENT COMPONENTS

### 6.1. Fourier velocity components of ions and electrons

Let us give velocities of ions and electrons in the Fourier-representation. From (20), (21), and (40), we have

$$v_{i1xk} = \frac{1}{\omega_{ci}^2} \left( 1 + \frac{\omega^2}{\omega_{ci}^2} \right) Q_{i1xk} + i \frac{k_y^2}{k_z^2} \frac{(\omega^2 - g_i a_i)}{\omega \omega_{ci}^3} Q_{i1yk} - \frac{1}{\omega_{ci}} \frac{k_y}{k_z} \left( 1 - i \frac{a_i}{k_z} \right) F_{i1zk}, \quad (66)$$

$$v_{i1yk} = \frac{1}{\omega_{ci}^2} \left[ 1 + \frac{(k^2 \omega^2 - k_y^2 g_i a_i)}{k_z^2 \omega_{ci}^2} \right] Q_{i1yk} + i \frac{\omega}{\omega_{ci}^2} \frac{k_y}{k_z} \left( 1 - i \frac{a_i}{k_z} \right) F_{i1zk}. \quad (67)$$

Here and below, we have introduced notations

$$a_{i,e} = \frac{1}{c_{si,e}^2} \left[ (\gamma - 1) g_{i,e} + \frac{\partial c_{si,e}^2}{\partial z} \right]. \quad (68)$$

The velocity  $v_{i1zk}$  is given by (38).

For electrons, using (64) and (65), we find

$$\begin{aligned} v_{e1xk} = & -i \frac{a_e c_{se}^2}{\omega \omega_{ci}} \frac{k_y^2}{k_z^2} \left( b_e \frac{1}{m_e \omega_{ce}} F_{e1xk} + \omega \frac{\partial T_{e0}}{k_z T_{e0} \partial z} \frac{B_{1yk}}{B_0} \right) \\ & + \frac{1}{m_e \omega_{ce}} F_{e1yk} - \frac{k_y}{k_z} \left( 1 - i \gamma \frac{a_e}{k_z} \right) \frac{1}{m_e \omega_{ce}} F_{e1zk}, \end{aligned} \quad (69)$$

where the following notation is introduced:

$$b_e = \frac{1}{c_{se}^2} \left( \gamma g_e + \frac{\partial c_{se}^2}{\partial z} \right). \quad (70)$$

Equation (60) gives us

$$v_{e1zk} = \frac{k_y}{k_z} \frac{1}{m_e \omega_{ce}} F_{e1xk} - i \frac{k_y}{k_z^2} \left( b_e \frac{1}{m_e \omega_{ce}} F_{e1xk} + \omega \frac{\partial T_{e0}}{k_z T_{e0} \partial z} \frac{B_{1yk}}{B_0} \right) - i \frac{\omega}{k_z^2 T_{e0}} \left( 1 + i \omega \frac{n_{e0}}{\chi_{e0} k_z^2} \right) F_{e1zk}. \quad (71)$$

The velocity  $v_{e1y}$  is defined by (52).

## 6.2. Fourier electron velocity components at the absence of the heat flux

To elucidate the role of the electron thermal flux, we will also consider the dispersion relation when this flux is absent. Therefore, we also give here the corresponding electron velocity components:

$$v_{e1xk} = -i \frac{k_y^2 g_e a_e}{k_z^2 \omega \omega_{ci} m_e \omega_{ce}} F_{e1xk} + \frac{1}{m_e \omega_{ce}} F_{e1yk} - \frac{k_y}{k_z} \left( 1 - i \frac{a_e}{k_z} \right) \frac{1}{m_e \omega_{ce}} F_{e1zk}, \quad (72)$$

$$v_{e1zk} = \frac{k_y}{k_z} \left( 1 - i \frac{g_e}{k_z c_{se}^2} \right) \frac{1}{m_e \omega_{ce}} F_{e1xk} - i \frac{\omega}{k_z^2 c_{se}^2 m_i} \left( 1 - i \frac{\gamma g_e}{k_z c_{se}^2} \right) F_{e1zk}. \quad (73)$$

Comparing expressions (69) and (71) with these equations, we see that the thermal flux essentially modifies the small terms in the electron velocity under conditions (57).

## 6.3. Fourier components of current

We find now the Fourier components of the linear current  $\mathbf{j}_1 = q_i n_{i0} \mathbf{v}_{i1} + q_e n_{e0} \mathbf{v}_{e1}$ . It is convenient to consider the value  $4\pi i \mathbf{j}_1 / \omega$ . Using expressions (38), (52), (66), (67), (69), and (71), we obtain the following current components:

$$\begin{aligned} \frac{4\pi i}{\omega} j_{1xk} &= a_{xx} E_{1xk} + i a_{xy} E_{1yk} - a_{xz} E_{1zk} \\ &\quad - b_{xx} (v_{i1xk} - v_{e1xk}) - i b_{xy} (v_{i1yk} - v_{e1yk}) + b_{xz} (v_{i1zk} - v_{e1zk}), \end{aligned} \quad (74)$$

$$\begin{aligned} \frac{4\pi i}{\omega} j_{1yk} &= -ia_{yx}E_{1xk} + a_{yy}E_{1yk} - a_{yz}E_{1zk} \\ &+ ib_{yx}(v_{i1xk} - v_{e1xk}) - b_{yy}(v_{i1yk} - v_{e1yk}) + b_{yz}(v_{i1zk} - v_{e1zk}), \end{aligned} \quad (75)$$

$$\begin{aligned} \frac{4\pi i}{\omega} j_{1zk} &= -a_{zx}E_{1xk} - a_{zy}E_{1yk} + a_{zz}E_{1zk} \\ &+ b_{zx}(v_{i1x} - v_{e1x}) + b_{zy}(v_{i1y} - v_{e1y}) - b_{zz}(v_{i1z} - v_{e1z}). \end{aligned} \quad (76)$$

When obtaining (74)-(76), we have used notations (16), (23), (24), and (48) and equalities  $q_e = -q_i$ ,  $n_{e0} = n_{i0}$ ,  $m_e\nu_{ei} = m_i\nu_{ie}$ . We also have substituted  $B_{1yk}$  by  $(k_z c/\omega)E_{1xk}$ , using (9). The following notations are introduced in (74)-(76):

$$\begin{aligned} a_{xx} &= \frac{\omega_{pi}^2 k^2}{\omega_{ci}^2 k_z^2} \left( 1 - \frac{k_y^2 g_i a_i + a_e b_e c_{se}^2}{k^2 \omega^2} - \frac{k_y^2 a_e c_{se}^2}{k^2 \omega^2} \frac{\partial T_{e0}^*}{T_{e0} \partial z} \right), \\ a_{xy} &= a_{yx} = \frac{\omega_{pi}^2 \omega k^2}{\omega_{ci}^3 k_z^2} \left( 1 - \frac{k_y^2 g_i a_i}{k^2 \omega^2} \right), \quad a_{xz} = \frac{\omega_{pi}^2 k_y}{\omega \omega_{ci} k_z^2} (a_i - \gamma a_e), \\ a_{yy} &= \frac{\omega_{pi}^2}{\omega_{ci}^2}, \quad a_{yz} = a_{zy} = \frac{\omega_{pi}^2 k_y}{\omega_{ci}^2 k_z}, \quad a_{zx} = \frac{\omega_{pi}^2 k_y}{\omega \omega_{ci} k_z} \left( b_e - \frac{g_i}{c_{si}^2} + \frac{\partial T_{e0}^*}{T_{e0} \partial z} \right), \\ a_{zz} &= \frac{\omega_{pi}^2}{k_z^2} \left( \frac{\gamma}{c_{se}^2} + \frac{1}{c_{si}^2} \right) \end{aligned} \quad (77)$$

and

$$\begin{aligned} b_{xx} &= \frac{\omega_{pi}^2 \nu_{ie}}{\omega_{ci}^2} \frac{m_i k^2}{q_i k_z^2} \left( 1 - \frac{k_y^2 g_i a_i + a_e c_{se}^2 b_e}{k^2 \omega^2} \right), \\ b_{zx} &= \frac{\omega_{pi}^2 k_y}{\omega \omega_{ci} k_z^2} \left( b_e - \frac{g_i}{c_{si}^2} \right) \frac{m_i}{q_i} \nu_{ie}, \\ b_{ij} &= a_{ij} \frac{m_i}{q_i} \nu_{ie}. \end{aligned} \quad (78)$$

Here  $\omega_{pi} = (4\pi n_{i0} q_i^2 / m_i)^{1/2}$  is the ion plasma frequency and  $k^2 = k_y^2 + k_z^2$ . The terms proportional to  $T_{e0}^*$  are connected with the background electron thermal flux.

Calculations show that to obtain expressions for  $a_{ij}$  without thermal flux, using electron velocities (72) and (73), we must change  $b_e$  by  $g_e/c_{se}^2$ , put  $T_{e0}^* = 0$ , and take  $\gamma = 1$  in terms  $a_{xz}$  and  $a_{zz}$ .

#### 6.4. Simplification of collision contribution

An assumption that electrons are magnetized has only been involved by neglecting the transverse electron thermal flux. In other respects, a relationship between  $\omega_{ce}$  and  $\nu_{ei}$  or  $\omega_{ci}$  and  $\nu_{ie}$  (that is the same) can be arbitrary in (74)-(76). We further proceed by taking into account that  $\omega \ll \omega_{ci}$ . In this case, we can neglect the collisional terms proportional to  $b_{xy}$  and  $b_{yx}$ . However, the system of equations (74)-(76) stays sufficiently complex to find  $\mathbf{j}_1$  through  $\mathbf{E}_1$ . Therefore, we further consider the case in which the frequency  $\omega$  and wave numbers satisfy the following conditions:

$$\frac{\omega_{ci}^2 k_z^2}{\nu_{ie}^2 k^2} \gg \frac{\omega}{\nu_{ie}} \gg \frac{1}{k_z^2 H^2} \frac{k_y^2 c_s^2}{\omega_{ci}^2}, \quad (79)$$

where

$$c_s^2 = \frac{c_{si}^2 c_{se}^2}{\gamma c_{si}^2 + c_{se}^2}. \quad (80)$$

It is clear that conditions (79) can easily be realized. In this case, the current components are equal to

$$\begin{aligned} \frac{4\pi i}{\omega} j_{1xk} &= \varepsilon_{xx} E_{1xk} + i\varepsilon_{xy} E_{1yk} - \varepsilon_{xz} E_{1zk}, \\ \frac{4\pi i}{\omega} j_{1yk} &= -i\varepsilon_{yx} E_{1xk} + \varepsilon_{yy} E_{1yk} - \varepsilon_{yz} E_{1zk}, \\ \frac{4\pi i}{\omega} j_{1zk} &= -\varepsilon_{zx} E_{1xk} - \varepsilon_{zy} E_{1yk} + \varepsilon_{zz} E_{1zk}. \end{aligned} \quad (81)$$

Components of the dielectric permeability tensor  $\varepsilon_{ij}$  are the following:

$$\begin{aligned} \varepsilon_{xx} &= a_{xx} + i \frac{\nu_{ie}}{\omega_{ci}} \frac{k_y}{k_z^2} \frac{(a_i - \gamma a_e)}{(1 - id_z)} a_{zx}, \varepsilon_{xy} = a_{xy} + \frac{\nu_{ie}}{\omega_{ci}} \frac{k_y}{k_z^2} \frac{(a_i - \gamma a_e)}{(1 - id_z)} a_{zy}, \\ \varepsilon_{xz} &= \frac{a_{xz}}{(1 - id_z)}, \varepsilon_{yx} = a_{yx} - \frac{\omega \nu_{ie}}{\omega_{ci}^2} \frac{k_y}{k_z} \frac{a_{zx}}{(1 - id_z)}, \varepsilon_{yy} = a_{yy}, \\ \varepsilon_{yz} &= \frac{a_{yz}}{(1 - id_z)}, \varepsilon_{zx} = \frac{a_{zx}}{(1 - id_z)}, \varepsilon_{zy} = \frac{a_{zy}}{(1 - id_z)}, \varepsilon_{zz} = \frac{a_{zz}}{(1 - id_z)}, \end{aligned} \quad (82)$$

where we have used notations (78). Parameter  $d_z$ ,

$$d_z = \frac{\omega \nu_{ie}}{k_z^2 c_s^2}, \quad (83)$$

defines the collisionless,  $d_z \ll 1$ , and collisional,  $d_z \gg 1$ , regimes. Below, we derive the dispersion relation.

## 7. DISPERSION RELATION

Using (9) and (10) in the Fourier-representation and a system of equations (81), we obtain the following equations for the electric field components:

$$\begin{aligned} (n^2 - \varepsilon_{xx}) E_{1xk} - i\varepsilon_{xy} E_{1yk} + \varepsilon_{xz} E_{1zk} &= 0, \\ i\varepsilon_{yx} E_{1xk} + (n_z^2 - \varepsilon_{yy}) E_{1yk} + (-n_y n_z + \varepsilon_{yz}) E_{1zk} &= 0, \\ \varepsilon_{zx} E_{1xk} + (-n_y n_z + \varepsilon_{zy}) E_{1yk} + (n_y^2 - \varepsilon_{zz}) E_{1zk} &= 0, \end{aligned} \tag{84}$$

where  $\mathbf{n} = \mathbf{k}c/\omega$ . The dispersion relation can be found by setting the determinant of the system (84) equal to zero. In our case, the terms proportional to  $\varepsilon_{xy}$  and  $\varepsilon_{yx}$  can be neglected. As a result, we have

$$\begin{aligned} (n^2 - \varepsilon_{xx}) [n_y^2 \varepsilon_{yy} + (n_z^2 - \varepsilon_{yy}) \varepsilon_{zz} - n_y n_z (\varepsilon_{yz} + \varepsilon_{zy}) + \varepsilon_{yz} \varepsilon_{zy}] \\ + (n_z^2 - \varepsilon_{yy}) \varepsilon_{xz} \varepsilon_{zx} = 0. \end{aligned} \tag{85}$$

This dispersion relation can be studied for different cases. In subsequent sections, we consider both the collisionless and collisional cases.

### 7.1. Collisionless case

We assume that condition

$$\frac{\omega \nu_{ie}}{k_z^2 c_s^2} \ll 1, \tag{86}$$



is satisfied. Then, using notations (77) and (82), we reduce the dispersion relation (85) to the form

$$(\omega^2 - k_z^2 c_A^2) \left( \omega^2 - k_z^2 c_A^2 - \Omega^2 \frac{k_y^2}{k^2} \right) = 0, \quad (87)$$

where  $c_A = B_0 / (4\pi m_i n_{i0})^{1/2}$  is the Alfvén velocity and

$$\Omega^2 = g_i a_i + c_{se}^2 a_e b_e + c_{se}^2 a_e \frac{\partial T_{e0}^*}{T_{e0} \partial z} + c_s^2 (a_i - \gamma a_e) \left( b_e - \frac{g_i}{c_{si}^2} + \frac{\partial T_{e0}^*}{T_{e0} \partial z} \right). \quad (88)$$

When obtaining (87), we have used the condition  $k_y^2 c_s^2 / \omega_{ci}^2 \ll 1$ . We see that there are two wave modes. The first mode,  $\omega^2 = k_z^2 c_A^2$ , is the Alfvén wave with a polarization of the electric field mainly along the  $y$ -axis (remind that the wave vector  $\mathbf{k}$  is situated in the  $y - z$  plane). This wave does not feel the inhomogeneity of medium. The second wave has a polarization of the magnetosonic wave, i.e. its electric field is directed mainly along the  $x$ -axis (see below). This wave is undergone by the action of the medium inhomogeneity effect. The corresponding dispersion relation is

$$\omega^2 = k_z^2 c_A^2 + \Omega^2 \frac{k_y^2}{k^2}. \quad (89)$$

Expression (88) can further be simplified, using (11), (12), (68), (70), and (80). As a result, we obtain

$$\Omega^2 = \frac{\gamma}{(\gamma c_{si}^2 + c_{se}^2) m_i^2} \left[ (\gamma - 1) m_i g + \gamma \frac{\partial (T_{i0} + T_{e0})}{\partial z} \right] \left[ m_i g + \frac{\partial (T_{e0} + T_{e0}^*)}{\partial z} \right]. \quad (90)$$

We have pointed out at the end of Sect. 6.3 what changes must be done in expressions (77) and (78) to consider the case without the electron heat flux. This case follows from (90), if we omit the term  $\partial (T_{e0} + T_{e0}^*) / \partial z$  and put  $\gamma = 1$  in the first multiplier. Then  $\Omega^2$  becomes ( $\Omega^2 \rightarrow \Omega_1^2$ )

$$\Omega_1^2 = \frac{g}{(c_{si}^2 + c_{se}^2)} \left[ (\gamma - 1) g + \frac{\partial (c_{si}^2 + c_{se}^2)}{\partial z} \right]. \quad (91)$$

This is the Brunt-Väisälä frequency. Comparing expressions (90) and (91), we see that the heat flux stabilizes the unstable ( $\Omega_1^2 < 0$ ) stratification. We also see from (90) that the background heat flux ( $\sim T_{e0}^*$ ) has no a fundamental importance. If the temperature decreases in the direction of gravity ( $\partial T_{i,e0}/\partial z > 0$ ), the medium is stable. Solution (90) describes an instability only when

$$\frac{\gamma - 1}{2\gamma} m_i g < -\frac{\partial T_0}{\partial z} < \frac{1}{2} m_i g,$$

where  $T_{i0} \sim T_{e0} = T_0$ . We also note that  $\Omega^2$  can be negative if gradients of  $T_{i0}$  and  $T_{e0}$  have different signs.

The dispersion relation (87) with  $\Omega^2$  defined by (90) considerably differs from the dispersion relation obtained in the framework of the ideal MHD (Quataert 2008). The reasons of this are discussed in Sect. 8.

## 7.2. Collisional case

We proceed with the collisional case when

$$\frac{\omega \nu_{ie}}{k_z^2 c_s^2} \gg 1. \tag{92}$$

In this limiting case, the dispersion relation takes the form  $n^2 - \varepsilon_{xx} \approx 0$  and we obtain again (89). Thus, the dispersion relation is the same for both the collisionless and collisional cases. We note that this result has also been obtained for the case in which gravity is perpendicular to the magnetic field (Nekrasov and Shadmehri 2010).

### 7.3. Polarization of perturbations

Let us neglect in the system of equations (84) the small contributions given by  $\varepsilon_{xy}$  and  $\varepsilon_{yx}$ . Then, for example, in the collisionless case, we obtain for the second wave  $\omega^2 \neq k_z^2 c_A^2$ ,

$$\begin{aligned} E_{1yk} &= \frac{k_y}{k_z} E_{1zk}, \\ E_{1zk} &= \frac{\varepsilon_{zx}}{\varepsilon_{zz}} E_{1xk} \ll E_{1xk}. \end{aligned} \tag{93}$$

Thus, the second wave has a polarization of the electric field mainly along the  $x$ -axis. In spite of that the component  $E_{1zk} \ll E_{1xk}$ , it is multiplied by a large coefficient in the first equation of the system (84). As a result, the contribution of this term is the same on the order of magnitude as that of the first term.

In the collisional case, the component  $E_{1zk}$  is also defined by (93). However, its contribution to the first equation of the system (84) can be neglected. However, the contribution of the collisional term connected with the longitudinal current in (74) which is proportional to  $E_{1xk}$  in (76) is important.

## 8. DISCUSSION

In the MHD analysis of the buoyancy instabilities, one assumes that  $p_{i1k}/p_{i0} \ll n_{i1k}/n_{i0}$  (see, e.g., Balbus 2000; Quataert 2008; Ren et al. 2009). This relation is correct for internal gravity waves in the neutral medium (e.g., Nekrasov 1994). It is also correct for perturbations  $n_{i1k}$  and  $p_{i1k}$  connected with the transverse perturbations  $Q_{i1yk}$

$$\frac{p_{i1k}}{p_{i0}} / \frac{n_{i1k}}{n_{i0}} (\sim Q_{i1yk}) \sim \frac{g_i}{k_z c_{si}^2} \ll 1$$

(see 41 and 42). However, it is followed from the last equations that due to the longitudinal electric field perturbation  $E_{1zk}$  which is of the order of  $E_{1zk} \sim (k_y c_s^2 / \omega \omega_{ci} H) E_{1xk}$  (see 77,

82, and 93) the relative pressure and number density perturbations are of the same order of magnitude

$$\frac{p_{i1k}}{p_{i0}} / \frac{n_{i1k}}{n_{i0}} (\sim E_{1zk}) \sim 1.$$

The ideal MHD does not capture the field  $E_{1z}$ . Therefore, results obtained in the MHD framework and multicomponent plasma approach are different. In Sect. 4.3, we also have shown that  $\nabla \cdot \mathbf{v}_{i1} \sim (g_i/c_{si}^2)v_{i1z}$ . This relation is also true for internal gravity waves (Nekrasov 1994). The fact that  $\nabla \cdot \mathbf{v}_{i1} \neq 0$  for gaseous media is taken into account when deriving an internal energy equation ( $\nabla \cdot \mathbf{v}_{i1}$  is excluded from the number density and pressure or temperature equations). Then in the MHD framework, one can use the condition of incompressibility  $\nabla \cdot \mathbf{v}_{i1} = 0$  in the momentum and magnetic induction equations for perturbations much slower than the sound waves. In our case, the divergence of the velocity defined by the main terms in  $\mathbf{v}_{i1k} \sim Q_{i1yk}$  (see 38 and 67) and  $\mathbf{v}_{e1k} \sim F_{e1xk}$  (see 52 and 71) is also equal to zero. However, these main terms are the same for ions and electrons and canceled with each other at calculation of the current. Therefore, together with velocity perturbations proportional to the longitudinal force  $F_{i,e1zk}$ , we must take into account contribution of additional small velocities connected with transverse perturbations  $Q_{i1x,yk}$  and  $F_{e1xk}$ .

From the dispersion relation (85), we see the necessity of involving the contribution of values  $\varepsilon_{xz}$ ,  $\varepsilon_{zx}$ , and  $\varepsilon_{zz}$  in the collisionless case (86) (values  $\varepsilon_{xz}$  and  $\varepsilon_{zx}$  give the last term on the right hand-side of 88). This means that contribution of currents  $j_{1x} \sim E_{1z}$  and  $j_{1z} \sim E_{1x}, E_{1z}$  must be taken into account. As for the collisional case (92), the electric field  $E_{1z}$  is not important. In the current  $j_{1xk}$ , we must take into consideration the contribution of the current  $j_{1zk}$  as a result of collisions which is proportional to  $E_{1xk}$  (see 74 and 76). This collisional case also is not captured by the ideal MHD.

Thus, the standard MHD equations with simplified assumptions are not applicable

for the correct theory of buoyancy instabilities. Such a theory can only be given by the multicomponent approach used in this paper.

The results following from (90) show that the thermal flux stabilizes the buoyancy instability in the case of the geometry under consideration. The instability only is possible in the narrow region of the temperature gradient (see Sect. 7.1). The presence of the background electron thermal flux (the term  $\sim T_{e0}^*$ ) does not play an essential role. An instability also is possible, if the temperature gradients of ions and electrons have the opposite signs.

The contribution of collisions between electrons and ions depends on the parameter  $d_z$  defined by (83). In the both limits (86) ( $d_z \ll 1$ ) and (92) ( $d_z \gg 1$ ), the dispersion relation has the same form.

In our analysis, we have considered for generality that ions and electrons have different temperatures. However in (3), (6), and (7), the terms describing the energy exchange between species due to their collisions has not been taken into account. This is possible, if the dynamical timescale is smaller than the timescale of smoothing of the ion and electron temperatures, i.e.  $\nu_{ie} \ll \Omega$ . In the opposite case,  $\nu_{ie} \gg \Omega$ , the perturbed temperatures of electrons and ions are almost equal one another. Equations (6) and (7) for electrons will keep their form because  $\mathbf{v}_{e1} \approx \mathbf{v}_{i1}$ . In the case  $T_{e0} \approx T_{i0}$ , these equations will stay the same with the heat flux two times less than the former one.

Conditions of our consideration (33) and (35) are satisfied when  $1 \gg \rho_i/H$  and  $1 \gg k_z H k_y^2 \rho_i^2$ , where  $\rho_i$  is the ion Larmor radius. For estimations, we take  $\omega \sim g/c_s$  ( $T_{e0} \sim T_{i0}$ ). It is obvious that these inequalities can be justified. It is easy to verify that conditions (79) are also satisfied. However, conditions (57) can impose some restrictions. In

the case  $T_{e0} \sim T_{i0}$ , they can be written in the form

$$1 \gg \frac{k_z c_s}{\nu_{ie}} \gg \frac{1}{k_z H}, \quad (94)$$

where we have used expression for  $\chi_{e0}$  (Braginskii 1965). From inequalities (94), it is in particular followed that the case of consideration is justified when  $\nu_{ie} \gg \omega$  and  $d_z \ll 1$ . In the case  $T_{e0} \gg T_{i0}$ , the value  $d_z$  is in the limits

$$\frac{T_{e0}}{T_{i0}} \gg d_z \gg \frac{T_{e0}}{T_{i0}} \frac{1}{k_z H}$$

and can be both  $< 1$  and  $\gtrsim 1$ .

We will further compare results obtained in this paper with the case when stratification is perpendicular to the magnetic field (Nekrasov and Shadmehri, 2010). However, first of all, we would like to say a few words about the Schwarzschild criterion of the buoyancy instability. It is generally accepted that this instability is possible, if the entropy increases in the direction of gravity. From a formal point of view, it is correct, if we take the Brunt-Väisälä frequency  $N$  in the form (e.g. Balbus 2000),

$$N^2 = -\frac{1}{\gamma \rho} \frac{\partial p}{\partial z} \frac{\partial \ln p \rho^{-\gamma}}{\partial z}.$$

However, this expression can easily be transformed into expression (32). Thus, we see that the buoyancy instability exists, if the temperature increases along the gravity and the temperature gradient exceeds a certain threshold.

When the thermal conduction is the dominant process in the electron temperature evolution, the buoyancy instability in the case  $g \perp B_0$  can arise according to criterion which is analogous to the Schwarzschild criterion (see 65 in Nekrasov and Shadmehri, 2010). The same is also true when the thermal conduction is negligible (66 in Nekrasov and Shadmehri, 2010). Both criteria are similar. Thus, we can conclude that from the point of view of observations it is difficult to define the role of thermal conduction in generation of instability

and turbulence. However, in the case  $g \parallel B_0$ , the situation is different. For the negligible electron heat flux, we have the similar criterion of instability as that for  $g \perp B_0$  (91). When the thermal conduction is dominant, the possible instability at  $\partial(T_{i0} + T_{e0})/\partial z < 0$  is stabilized (90). These results could be used by observers for determination of the mutual orientation of the magnetic field and gravity in astrophysical objects, e.g., in galaxy clusters.

The true geometry of the magnetic field lines in the ICM is poorly understood. However, one may consider two extreme cases for the direction of the dominant magnetic field lines depending on the direction of gravity. The direction of the latter can have a vital role in driving turbulence via the possible effect of convective heat flux. Since this flux is mainly along the magnetic field lines, the two extreme cases are considered as either the magnetic field is perpendicular to the gravity or parallel to it. Thus, the true response of ICM to small perturbations would be possibly between these two cases. Of course, when the system evolves and enters into the nonlinear regime, one may expect saturation of the instability by rearranging the magnetic field lines. Measurements of the magnetic field in ICM are not consistent and there is a factor of four to ten of discrepancy depending on the method (e.g., Carilli and Taylor 2002). Physical mechanisms that may affect these observational measurements of the magnetic field in ICM are very important in this regard. Our analysis gives a better understanding of such a mechanism, i.e. buoyancy instability, though more detailed work is needed in future.

## 9. CONCLUSION

In this paper, we have investigated buoyancy instabilities in magnetized electron-ion plasmas with the anisotropic electron thermal flux, using the multicomponent approach

when the dynamical equations for the ions and electrons are solved separately via electric field perturbations. We have included the background electron heat flux and collisions between electrons and ions. The important role of the longitudinal electric field perturbation, which is not captured by the ideal MHD equations, has been demonstrated. We have shown that the previous MHD result for the growth rate in the geometry considered in this paper when all background quantities are directed along the one axis is questionable. The reason of this has been shown to be in simplified assumptions made in the MHD analysis of the buoyancy instabilities and some shortcomings of the MHD.

At the consideration of the electron heat flux, we have adopted that the electron cyclotron frequency is much larger than the electron collision frequency that is typical for tokamaks, solar corona, and astrophysical objects such as ICM and galaxy clusters. The dispersion relation obtained shows that the anisotropic electron heat flux including the background one stabilizes the unstable stratification except the narrow region of the temperature gradient. However, when gradients of the ion and electron temperatures have opposite signs, the medium becomes unstable.

Results obtained in this paper are applicable to the magnetized stratified objects and can be useful for searching sources of turbulent transport of energy and matter. For astrophysical plasmas, it has been suggested that the buoyancy instability can act as a driving mechanism to generate turbulence in ICM and this extra source of heating may help to resolve the cooling flow problem (e.g., Allen 2000). However, all previous analytical and numerical studies are restricted to the MHD approach. Our study shows that when the true multifluid nature of the system with the electron heat flux is considered, one can not expect the buoyancy instability unless for a very limited range of the gradient of the temperature or when the gradients of the temperature of the electrons and ions have opposite signs. Both cases are very unlikely. However, in the case when the heat flux does not play the



role, the system can be unstable due to the convective instability.

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